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Quantitative validation and comparison of a range of forest growth model types

Guy Pinjuv [a,*], Euan G. Mason [a], Mike Watt [b]

[a] University of Canterbury, Private bag 4800, Chirstchurch, New Zealand
[b] Forest Research, New Zealand

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Abstract

Predictions from a range of model types (simplified process-based, a statistical state space, statistical difference, and a hybrid model) were compared to 969 measurements of forest growth across an environmental gradient. The models compared were 3-PG, CANTY, CanSPBL(1.2), and CanSPBL(water). The study made an objective comparison and validation of model types, with the main criterion for comparison being each model's ability to match actual historical measurements of forest growth in an independent data set. A number of stand level forest growth variables were compared including basal area, mean top height, and stocking over 14,058 ha of plantation-grown *Pinus radiata* in south-eastern New Zealand. Stand variable predictions at 195 permanent plot locations covering a range of elevations from 0 to 660 m were highly correlated with field estimates derived from plot data. The hybrid model CanSPBL(water) on average was the most accurate model in the study where predictions of stocking, basal area, and mean top height were 96%, 96%, and 96% efficient. The statistical-difference equation model CanSPBL(1.2) was equally efficient but on average 3% less accurate and slightly more biased in predictions suggesting that the hybrid model explained differences in growth due to differences water availability and soil type. The process-based model 3-PG predicted stocking and basal area 89% and 88% efficiently. Finally, the statistical state-space model CANTY predicted stocking, basal area, and mean top height 96%, 87%, and 87% efficiently. Results quantify the amount of precision that can be expected from the three model types, and suggest that each approach has strengths and weaknesses.

Keywords: Forest growth model; Physiological processes; Hybrid model; Forest productivity; Model validation

1. Introduction

Forestry growth models can be categorised according to their level of mechanistic detail, complexity, and generality (Battaglia and Sands, 1998). At one end of the scale are empirical growth and yield models, which are usually curves fitted to historical data describing forest growth from a particular region. This is possibly the simplest method of predicting forest growth assuming future growing conditions and climatic variables remain constant. However, such models are often limited in their general applicability and do little to elucidate the mechanisms of tree growth.

In contrast, process-based models are based on the mechanisms which underlie growth. As these models are responsive to changes in environmental and site conditions they are more generally applicable than empirical models. However, these models are seldom used as practical tools in forest management as they include many uncertainties and often require complex parameter values which are difficult to obtain (Mäkelä et al., 2000). Despite this, there has recently been much interest in applying process-based models to management applications, using simplified relationships developed from more detailed models (Landsberg and Waring, 1997).

Hybrid models or models that are a mix of process-based and empirical models can avoid the shortcomings of both approaches to some extent. These models incorporate a mechanistic description of the environmental influences into an empirical growth and yield model. Hybrid models provide an increase in biological realism over traditional empirical growth models, yet do not require the level of parameterisation of process-based models (Mäkelä et al., 2000).

Over the past decade there has been much debate within the scientific community on the relative merits of modelling forest growth with empirical-statistical versus process-based

* Corresponding author. Tel.: +64 3 364 2987; fax: +64 3 364 2124.
E-mail address: glp24@student.canterbury.ac.nz (G. Pinjuv).

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approaches. Many model comparisons such as Tickle et al. (2001) have dealt with only two model types and have included limited datasets across an also limited environmental gradient. No detailed comparisons of the three model types have been made across an environmental gradient to date. *Pinus radiata* is the most widely grown plantation species in the southern hemisphere and New Zealand. Within the Canterbury region of south-eastern New Zealand this species occurs across a broad environmental gradient that covers coastal forests, dry plains sites, to productive foothills sites ranging in elevation from 0 to 660 m. Given the environmental range which occurs in the this region, utilisation of data from a regional permanent sample plot network provides an ideal means of comparing growth estimates using empirical, hybrid, and process-based approaches.

The objective of this study is to make a comparison and validation of a range of model types, with the main criterion for comparison being each model's ability to match actual historical measurements of forest growth in an independent data set. The validation will use an extensive dataset that has been collected across an environmental gradient. The models compared were CANTY (Goulding, 1995), CanSPBL(1.2) (Pinjuv, 2006), CanSPBL(water) (Pinjuv, 2006), and 3-PG (Landsberg and Waring, 1997). CANTY (Goulding, 1995), is an empirical state-space stand level growth and yield model for radiata pine plantations in the Canterbury region of New Zealand. CanSPBL(1.2) (Pinjuv, 2006) is a difference method stand level growth and yield model for radiata pine plantations growing in Canterbury. CanSPBL(water) (Pinjuv, 2006) is a hybrid version of CanSPBL(1.2) (Pinjuv, 2006) with an index for root zone water balance over the growth interval incorporated into the model. Finally, 3-PG (Landsberg and Waring, 1997), is a simplified process-based growth model that can be adapted to a range of forest species by parameterisation of model coefficients.

2. Methods

The models were examined quantitatively, assessing model behaviour with a validation data set that had not been used for model construction. The procedure involved graphical displays and statistical tests. Potential correlation was detected by inspection of graphical plots of residual versus predictions and explanatory variables. Model residual errors or the difference between predictions made with the various models and observations of growth with the current data will display certain trends along with initial conditions, projection length, or predicted values when the model is biased. 3-PG was the only model evaluated that required a calibration specific to the study area before it could be validated, though it can be argued that fitting sigmoid equations to local PSP datasets may also represent local parameterisation. 3-PG was calibrated using an independent data set and varying select input parameters of the model to give good fits between measured and modelled variables. The climate data used in calibration and fitting of the models 3-PG, and CanSPBL(water) may not have been entirely independent of that used in the model validation from a statistical standpoint, as the climate data used interpolated weather measurements over a 10-year period.

No statistical tests are given in this analysis as repeated measurements have been taken from the basic experimental units (PSP). The consequences of this are: (i) estimators of the regression coefficients may no longer have minimum variance but will still be unbiased and consistent; (ii) standard errors of coefficients in the regression will be underestimated; (iii) any significance tests or confidence limits constructed using t or F distributions are likely to be incorrect since assumed independence of errors is violated (West et al., 1984). The mean square error (MSE) for the regression is also likely to be underestimated if the correlation is positive and inflated if the correlations are negative (Snowdon et al., 1999).

2.1. Models compared

2.1.1. Overview of CANTY

The statistical growth and yield model CANTY is a stand level model including the components of mean top height, basal area/ha, stems/ha and volume/ha. The model was built by the New Zealand Forest Research Institute (1991) and was intended for modelling growth and yield of radiata pine growing in the Canterbury region (Goulding, 1995). The model requires five stand level inputs for a simulation that are listed in Table 1. CANTY is a state-space model, that is a variant of the modelling approach outlined by Garcia (1994) were a state is defined by several state vectors. The state of a stand in such models is usually expressed by mean top height, basal area, stems/ha, and sometimes crown closure. Future states can be predicted by future management options and current states that are summaries of past growth. The behaviour of the system is described by a transition function and an output function. The output function for volume is predicted with state variables. The state-space approach is critically dependent on site index, which is estimated with a height function, which in turn can be highly dependent on the measurement intervals used in model fitting.

Some theoretical aspects of the model are described by the following:

$$\text{transition function}: \quad X(t) = F[X(t_0), U, T - T_0]$$

$$\text{or} \quad \frac{\mathrm{d}X}{\mathrm{d}t} = f(X) \tag{1}$$

$$\text{output function}: \quad V(t) = G[X(t)] \quad \text{or} \quad \frac{\mathrm{d}V}{\mathrm{d}t} = g(X) \tag{2}$$

where $X = (H, G, N)$, H = mean top height, G = basal area, N = stocking, V = volume, U = input (management options), T_0 = starting time of a period, and T = the end of a period of time.

When a multivariate generalisation of the Bertlanffy–Richards model is adopted, the new state vectors become

$$Y = (y_1, y_2, y_3) = (H^{C_{11}}, H^{C_{21}} G^{C_{22}} N^{C_{23}}, H^{C_{31}} G^{C_{32}} N^{C_{33}})$$

$$\text{or} \quad Y = X^C \tag{3}$$

Table 1
Model inputs for CANTY, CanSPBL(1.2), CanSPBL(water), and 3-PG (3-PG calibration parameters are listed in Appendices A1 and A2)

Stand information	Monthly climate	Soil
CANTY inputs		
Initial age (years)		
Final age (years)		
Initial height (m)		
Initial basal area (m^2 ha^{-1})		
Initial stocking (stems/ha)		
CanSPBL(1.2) inputs		
Initial age (years)		
Final age (years)		
Initial height (m)		
Initial basal area (m^2 ha^{-1})		
Initial stocking (stems/ha)		
Elevation (m)		
CanSPBL(water) inputs		
Initial age (years)	Rain (mm)	Soil type (3-PG soil classification)
Initial height (m)	Average temperature (°C)	Initial available soil water (mm)
Final age (years)	Maximum temperature (°C)	Maximum available soil water (mm)
Initial basal area (m^2 ha^{-1})	Minimum temperature (°C)	Minimum available soil water (mm)
Initial stocking (stems/ha)	Solar radiation (MJ/m^2)	
Elevation (m)	Vapour pressure (kPa)	
Initial leaf area index (m^2 m^{-2})		
3-PG inputs (listed below and 51 parameter inputs listed in Appendix)		
Planting date	Rain (mm)	Soil type (3-PG soil classification)
Initial age (years)	Average temperature (°C)	Initial available soil water (mm)
Final age (years)	Maximum temperature (°C)	Maximum available soil water (mm)
Initial height (m)	Minimum temperature (°C)	Minimum available soil water (mm)
Initial basal area (m^2 ha^{-1})	Solar radiation (MJ/m^2)	
Initial stocking (stems/ha)	Vapour pressure (kPa)	
Initial leaf area index (m^2 m^{-2})		
Initial weight of foliage (tonnes)		
Initial weight of roots (tonnes)		
Initial weight of stems (tonnes)		
Latitude		

The linear differential equation is

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = AY + b = AX^C + b \quad \text{or} \quad \frac{\mathrm{d}Y}{\mathrm{d}t} = A(X^C - a) \quad \text{when } a = A^{-1}b \tag{4}$$

where A and C is a 3×3 matrix, a and b is a 1×3 matrix, and $T = b_h t$ is scaled time, b_h is the coefficient in the site index equation.

A global difference equation:

$$X(t_2) = \{a + P^{-1}\,\mathrm{e}^{\Lambda b_h(t_2 - t_1)} P[X^C(t_1) - a]\}^{1/C} \tag{5}$$

is obtained from integration of the above differential equation. P and Λ are such that Λ is diagonal and $A = P^{-1}\Lambda P$.

A maximum likelihood estimator was used to estimate all equations simultaneously. The simultaneous estimation of all model components minimised overall errors but restricted the choice of functional form for individual state variables.

The model was prepared for testing by using a Basic code version provided by the Growth Modelling Research Cooperative. The code was adapted by Dr. E.G. Mason so that it could operate in batch mode.

2.1.2. Overview of 3-PG

The hybrid growth model 3-PG (physiological principles predicting growth) developed by Landsberg and Waring (1997), is a simplified process-based, stand level model of forest growth. This model expresses gross primary productivity (GPP) as the product of radiation-use efficiency (ε) and absorbed photosynthetically active radiation (APAR) (Eq. (6)). A set of modifiers reduce the efficiency of a unit of radiation as a result of soil water deficit (f_θ), vapour pressure deficit of the air (f_D), temperature (f_T), soil nutrition (f_N), and stand age (f_A) (Landsberg and Waring, 1997), where the minimum of f_N and f_D is used. Net primary production (NPP) is calculated as a fixed amount of GPP(c) and the general equation can be expressed as

$$\mathrm{NPP} = \mathrm{GPP}\,c = \left(\varepsilon \sum_{t-1}^{T} \mathrm{APAR}_t\, f_\theta\, f_D\, f_T\, f_N\, f_A\right) c \tag{6}$$

The 3-PG model consists of five simple process-based sub-models: the assimilation of carbohydrates, the distribution of biomass between foliage, roots and stems, the determination of stem number, soil water balance, and the conversion of biomass values into variables of interest to forest managers.

The model is run on a monthly time step and the state of the stand is updated every month over the simulation period. The model requires input parameters that describe the growth characteristics of the species, site characteristics, and climatic inputs (listed in Table 1, and Appendices A1 and A2). The 3-PG model predicts the time-course of stand development, water use, and available soil water. Its primary output variables are net primary production, the standing biomass in foliage, stem, and roots, stem numbers, available soil water, and transpiration. The model also outputs leaf area index, mean stem diameter at breast height, main stem volume, and mean annual increment. The tested model (Version 2.3) was provided by the Commonwealth Scientific and Industrial Research Organisation as Basic code within a Microsoft Excel spreadsheet. Visual Basic code was adapted to run the model in batch mode on monthly climatic inputs that displayed yearly variation.

The 3-PG model (Landsberg and Waring, 1997) requires monthly average climatic inputs of solar radiation, mean air temperature, atmospheric vapour pressure deficit, rainfall, and frost days. If mean (T_a), maximum (T_x), and minimum (T_n) air temperatures are known, then $T_a = 1/2(T_x + T_n)$. Vapour pressure deficit can also be estimated by the model from T_x and T_n, as half the difference between the saturated vapour pressure at T_x and T_n. The 3-PG model can be run using either actual monthly weather data or long-term monthly averages. Other inputs are variables describing the physical properties of the site and initial biomass pools, latitude, an unit-less site fertility rating, maximum available soil water, minimum available soil water, initial available soil water, initial weight of foliage, initial weight of stems, foliage, and roots, and a general descriptor of the soil texture (Landsberg and Waring, 1997).

Above ground initial biomass pools input into 3-PG were estimated using existing tree-level biomass equations for New Zealand (Moore, 2005) and are listed in Pinjuv (2006). Available soil water (ASW) levels and soil classification data were extracted from a local database of New Zealand soil types (Barringer et al., 1998) where soil attributes are linked to latitude and longitude. Soil classes were translated from the soils database format in Barringer et al. (1998) to 3-PG soil classes by referring to the soil texture triangle in McLaren and Cameron (1996). Initial stand state variables of latitude, year planted, month planted, initial year, initial month, end age, and initial stocking are available from the database used for model validation previously discussed.

The climatic inputs of solar radiation, minimum air temperature, maximum air temperature, and rainfall were estimated for each site using the method of correcting 10-year average values for each site by local met station measurements. Actual inputs are based on a system of corrected estimates from BIOCLIM (Leathwick and Stephens, 1998) climate surfaces. BIOCLIM is a set of surface equations for New Zealand that estimate climatic variables on a 10-year average based on interpolation between climate station measurements (Leathwick and Stephens, 1998). BIOCLIM outputs monthly averages of temperature, wind, rainfall, radiation, and average humidity extremes across New Zealand based on location. To simulate the monthly variation in actual climate readings, as opposed to mean monthly values over a 10-year period, the long-term mean monthly climate values derived from BIOCLIM were rescaled using monthly climate measurements from a nearby weather station. These estimates were corrected by reference point measurements made at Christchurch Airport (S43.5°, E172.55°, elevation 37 m). Differences between climatic estimates from BIOCLIM and measurements at Christchurch Airport were calculated for each month included in the study from 1984 to 2004. Corrections were added to long-term monthly estimates at each permanent sample plot. So instead of using the same monthly averages every year, a new value was calculated every month based on actual climate at Christchurch Airport to reflect years of extreme climatic events such as droughts. Similar approaches to correcting long-term averages with local measurements have been used by Tickle et al. (2001) and Snowdon et al. (1999).

Parameterisation of the model was achieved by: (1) adjusting selected parameters to obtain good fits of model output to observed measurements of stand basal area and stocking on a subset of calibration plots and (2) through the examination of time series estimates of leaf area index from the 3-PG model against estimates produced by model 1 presented in Pinjuv (2006). The calibration data set comprised 1059 growth measurements (independent of the validation dataset) taken within 200 unique plots that were representative of all elevation ranges, age classes, and re-measurement intervals in the study area. This method of parameterisation is similar to that used by Sands and Landsberg (2002), for stands of plantation-grown *Eucalyptus globulus* in Tasmania and Western Australia. Model calibration is described in detail in Pinjuv (2006), and a final set of parameter inputs are listed in Tables A1 and A2 in the Appendix. Calibration residual plots shown in Fig. 1 show no apparent bias.

2.1.3. Overview of CanSPBL(1.2)

The model CanSPBL(1.2) is a non-linear least-squares regression system of equations to predict forest growth. The model makes stand level predictions of mean top height (MTH), basal area/ha, stems/ha, volume/ha and diameter distribution. A simulation requires six stand level inputs listed in Table 1. Equations for each predicted variable were fitted using non-linear least-square regression procedures as described in detail in Pinjuv (2006). Diameter class distributions are described in the model using a reverse Weibull function. Stand tables can be produced with CanSPBL(1.2) using a recovery method of parameters to project future stand statistics, and finally the method of moments is used to convert stand statistics of standard deviation, maximum value, and arithmetic mean diameter to Weibull distribution parameters.

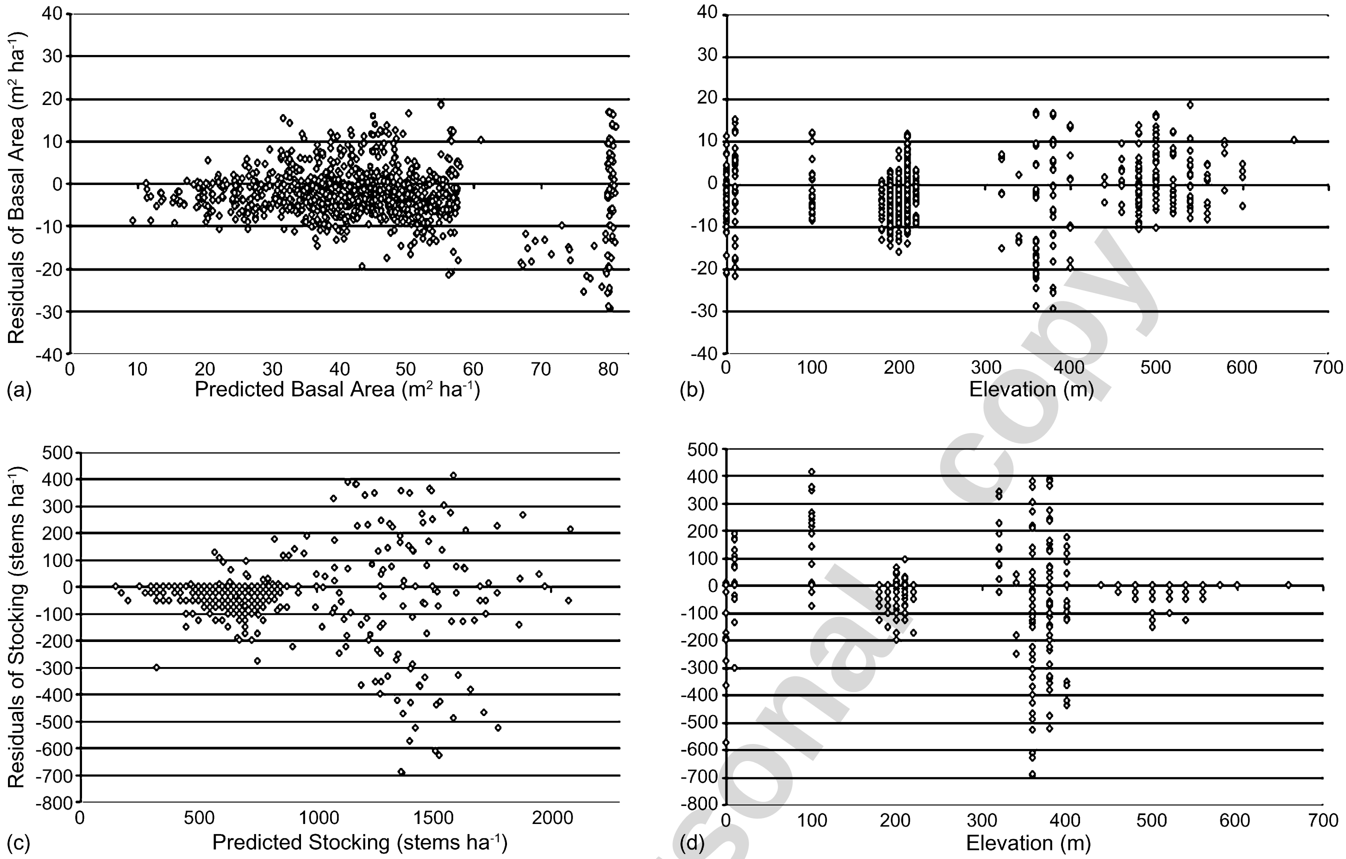

Fig. 1. Calibration residual plots for 3-PG showing: (a) basal area (G) residuals vs. predicted, (b) residuals of G vs. elevation, (c) residuals of stocking vs. predicted, and (d) residuals of stocking vs. elevation.

2.1.4. Overview of CanSPBL(water)

The model CanSPBL(water) is a non-linear least-squares regression system of equations to predict forest growth that includes the effects of available soil water over the simulation period. Available soil water predictions within CanSPBL(-water) are estimated with the sub-model for water balance from the process-based growth model 3-PG (Landsberg and Waring, 1997). The modelling approach used was to create a stand level model, where a simulation requires the seven stand level inputs, six monthly climate inputs, and four soil inputs listed in Table 1. Predictions made by the model are mean top height (MTH), basal area/ha, stems/ha, volume/ha and diameter distribution. Equations for each predicted variable were fitted using non-linear least-square regression procedures as described in Pinjuv (2006). Estimates of available soil water deficit are used within each equation to vary the asymptotic parameter of the difference equation. Diameter class distributions are also described in the model using a reverse Weibull function, and stand tables can be produced with the method of moments as described for CanSPBL(1.2). Both CanSPBL(1.2) and CanSPBL(water) were implemented in Microsoft Excel as code written for the purpose of independent validation.

2.2. Description of the validation dataset

The validation dataset comprised 964 growth observations taken within 193 unique permanent sample plots. All measurements were recorded between December 1982 and January of 2004 on the Selwyn Plantation Board Estate located in the Canterbury region of south-eastern New Zealand. Sites on the Selwyn estate vary with elevation covering the three forest types of plains, hills, and coastal sands with elevations ranging from 2 to 600 m above sea level. Distinctions between these forest types are due to both elevation and soil type. With increasing distance from the sea soils underlying forests in Canterbury change from coastal sands, shallow and dry floodplains soils, to deep wet loess hill soils (Barringer et al., 1998). Average temperatures are 10.9 °C for plains and coastal forests, and 11.6 °C for forests growing on the hills. Annual rainfall increases from 600 to 1100 mm for coastal-plains and hills forest, respectively (Leathwick and Stephens, 1998). The mean re-measurement interval is 6.6 years for all measurements, while the mean re-measurement interval per plot was 5.7 years. All validation data were independent of any information used for fitting or calibrating any of the included models in this study. A summary of plot variable estimates with a breakdown for plains and foothills plots is listed in Table 2, where the distinction between plains and foothills is 250 m elevation.

2.3. Validation tests

Model validation procedures in this study involved assessing model behaviour with the validation data set. Models were

Table 2
Summary of data for model validation

	Units	Mean	S.D.	Minimum	Maximum
Variable: plains data					
Age (initial)	Years	11.5	3.8	7.5	25.5
Age (final)	Years	18.0	5.2	7.7	30.4
Stocking (initial)	Stems/ha	668.1	143.0	275.0	1600.0
Stocking (final)	Stems/ha	645.0	137.4	275.0	1600.0
Mean top height (initial)	m	12.4	3.7	7.2	25.9
Mean top height (final)	m	19.1	4.9	8.7	29.3
Basal area (initial)	$m^2\ ha^{-1}$	19.1	11.4	4.6	61.6
Basal area (final)	$m^2\ ha^{-1}$	35.1	13.6	7.2	73.1
Dbh Std. D (initial)	mm	28	12	8	82
Dbh Std. D (final)	mm	44	15	9	87
Dbh Max. (initial)	mm	234	63	130	482
Dbh Max. (final)	mm	334	72	157	505
Elevation	m	135.8	60.1	0.0	250.0
Variable: hills data					
Age (initial)	Years	12.9	4.6	7.6	25.5
Age (final)	Years	19.6	5.7	9.6	29.4
Stocking (initial)	Stems/ha	823.2	424.0	250.0	2100.0
Stocking (final)	Stems/ha	754.7	362.7	250.0	1975.0
Mean top height (initial)	m	13.0	5.2	6.5	30.8
Mean top height (final)	m	21.1	6.8	8.4	33.9
Basal area (initial)	$m^2\ ha^{-1}$	34.7	26.9	4.9	104.0
Basal area (final)	$m^2\ ha^{-1}$	59.7	27.0	9.2	123.0
Dbh Std. D (initial)	mm	41	18	19	122
Dbh Std. D (final)	mm	63	24	22	131
Dbh Max. (initial)	mm	296	96	165	636
Dbh Max. (final)	mm	434	100	176	693
Elevation	m	455.6	85.6	290.0	660.0

Data shown represents 726 growth measurements taken from 131 unique plots located in the plains, and 238 growth measurements taken from 62 unique hills plots.

examined quantitatively using graphical displays and statistical tests of average model bias (AMB) and model efficiency (EF) (Loague and Green, 1991). Potential correlation was detected by inspection of graphical plots of residual versus predictions and explanatory variables for each of the main model components (mean top height, basal area, and stocking). Model residual errors or the difference between predictions made with final model equations and observations were examined for trends that indicated bias. Model performance has been described using AMB and EF (Loague and Green, 1991; Zhao, 1999). Average model bias (Eq. (7)) which is an average of errors for all predictions is described by

$$\mathrm{AMB} = \frac{1}{n}\sum(Y_i - \hat{Y}_i) \qquad (7)$$

where Y_i is the observed, $\hat{Y}_i$ is the modelled, and where $\bar{Y}_i$ is the average observed value. An AMB of 0 would indicate a model with no bias. Model efficiency (Eq. (8)) is also a measure of model performance and is described by

$$\mathrm{EF} = 1 - \frac{\sum (Y_i - \hat{Y}_i)^2}{\sum (Y_i - \bar{Y}_i)^2} \qquad (8)$$

where Y_i observed, $\hat{Y}_i$ is the modelled, and where $\bar{Y}_i$ is the average observed value. A high value of EF (maximum value of 1) indicates a model of perfect fit, while EF value of 0 indicates a model of poor fit where the average value would model the relationship as well. A negative EF value indicates an even poorer fit than the average value.

Dependent variables tested for bias and fit were chosen from those outputs which were common among models, and those that were viewed as useful to forest managers. These included mean top height (MTH), basal area (G), and the number of stems/ha (stocking). For each of these outputs, residual values were plotted against predicted value, elevation, time increment (projection interval), and initial value. Graphical output in this study was confined to only residual plots against elevation, as all other plots exhibited little apparent bias.

3. Results

3.1. CANTY validation

Model components of mean top height, basal area, and final stocking were validated by the display of graphical residual plots. Plots displayed residual values against elevation (Fig. 2). Fitting statistics of mean square error (MSE), average model bias (AMB), and model efficiency factor (EF) were also calculated, and are listed in Table 3.

CANTY showed severe bias in both mean top height and basal area against elevation, these effects were apparent in residual plots against elevation (Fig. 2a and b). However, residual plots of stocking showed little bias against elevation (Fig. 2c). The model CANTY was originally fit with data at

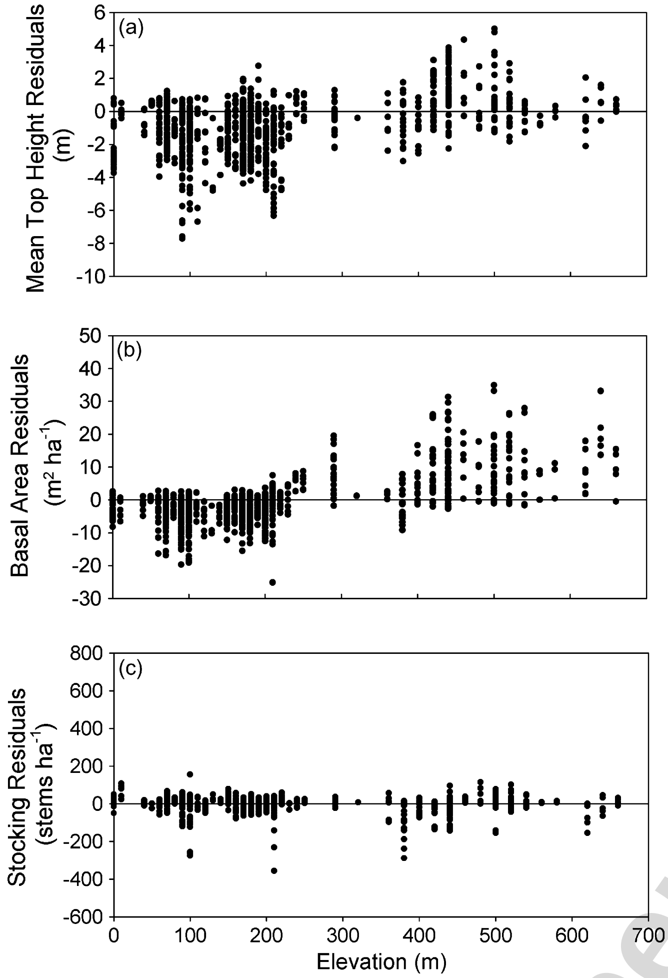

Fig. 2. Residual plots for CANTY validation showing: (a) mean top height residuals vs. elevation, (b) basal area residuals vs. elevation, and (c) stocking residuals vs. elevation.

Table 3
Statistics of model fit for validation datasets against CanSPBL(1.2), CANTY, 3-PG, and CanSPBL(water) in terms of mean square error (MSE), average model bias (AMB), and model efficiency factor (EF)

Model	Statistics of model fit		
	MSE	AMB	EF
CanSPBL(1.2)			
Stocking	2161.33	6.51	0.96
Basal area	17.99	0.55	0.96
Mean top height	1.11	0.08	0.96
CANTY			
Stocking	2022.45	−3.17	0.96
Basal area	55.00	−0.22	0.87
Mean top height	3.81	−0.91	0.87
3-PG			
Stocking	5566.82	−17.94	0.89
Basal area	53.48	1.08	0.88
CanSPBL(water)			
Stocking	2100.67	1.94	0.96
Basal area	17.32	0.46	0.96
Mean top height	1.07	0.06	0.96

lower elevations and with data of shorter average intervals than the data used for validation in this study. Because state-space models are critically sensitive to site index the severe bias in mean top height and basal area plots may have been due to the fact that that the model was fit with short interval data from plains elevations only.

3.2. Validation of 3-PG

Model components of basal area, and final stocking were validated by the display of graphical residual plots. Plots display residual values against elevation (Fig. 3a and b). Fitting statistics of mean square error, average model bias, and model efficiency factor were also calculated, and are listed in Table 3.

Residual plots indicate bias for basal area projection at higher elevation plots (Fig. 3a) that was not apparent in the calibration of the model (Fig. 1b). Basal area bias may also indicate an inability for 3-PG to deal with calibration on such a large scale without fine tuning all parameters for fitting such as the photosynthetic efficiency (alpha). Fitting the parameter alpha for each of the three elevation classes may have improved the calibration and validation fit. This parameter was not fit for the three forest types as they all contain the same species which was assumed to have the most influence over photosynthetic efficiency.

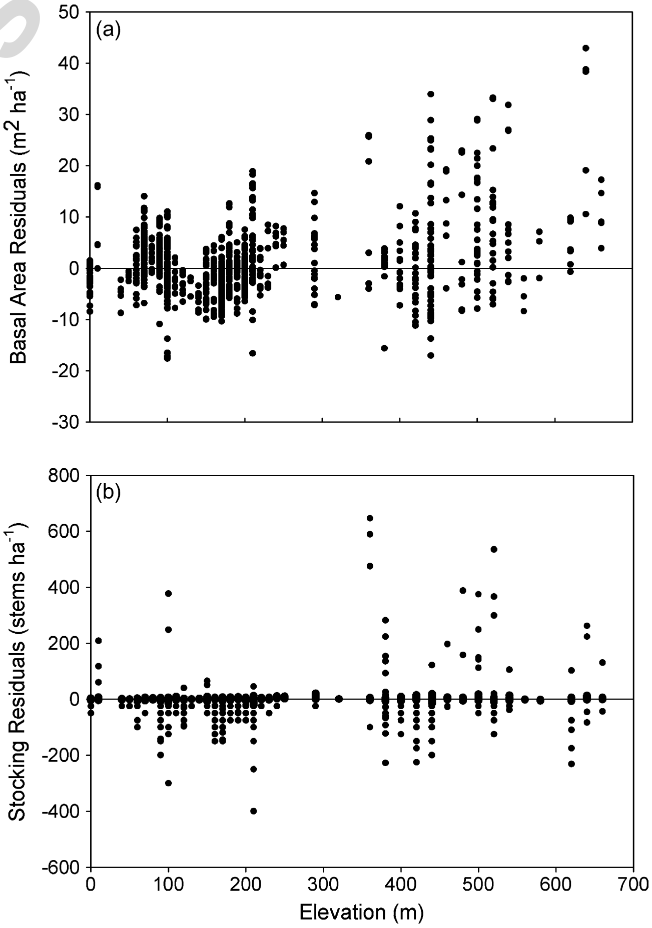

Fig. 3. Residual plots for 3-PG validation showing: (a) basal area residuals vs. elevation and (b) stocking residuals vs. elevation.

3-PG also showed an inability to deal with the prediction of stocking at the end of the simulation. This may have been due to mortality events not associated with overcrowding, as 3-PG uses the −3/2 power law alone to estimate mortality. Bias in mortality prediction is apparent in both the calibration residual plots (Fig. 1c and d) and validation plots (Fig. 3b), although the bias is more severe in the validation.

3.3. CanSPBL(1.2) validation

Model components of mean top height, basal area, and final stocking were validated by the display of graphical residual plots. Plots display residual values against elevation (Fig. 4). Fitting statistics are listed in Table 3.

Residual plots of mean top height and basal area showed little bias. The basal area model indicated possible bias in under prediction between elevations of 250 and 350 m (Fig. 4a). Stocking residuals indicate bias at plots above 450 m (Fig. 4c), where the model is underestimating stocking or at higher elevations. Stocking bias may have been an artefact of the way the mortality model was fitted in CanSPBL(1.2) to make predictions that seemed reasonable to managers. This process involved segregating the dataset to only include non-catastrophic mortality events so that mortality would not be overestimated on average plots. The validation dataset was not segregated in this way and residual bias at higher elevations may be due to catastrophic mortality events.

3.4. CanSPBL(water) validation

Model components of mean top height, basal area, and final stocking were validated by the display of graphical residual plots. Plots display residual values against elevation (Fig. 5). Fitting statistics were also calculated and are listed in Table 3.

Residual plots of mean top height and basal area showed little bias. The basal area model indicated possible bias in under prediction between elevations of 250 and 350 m (Fig. 5a). Stocking residuals indicate bias for plots located above 450 m (Fig. 5c). Stocking bias may have also been an artefact of the way the mortality model was fitted in CanSPBL(water) to make predictions that seemed reasonable to managers. This process involved segregating the dataset to only include non-catastrophic mortality events so that mortality would not be overestimated on average plots. The validation dataset was not segregated in this way and residual bias at higher elevations may be due to catastrophic mortality events.

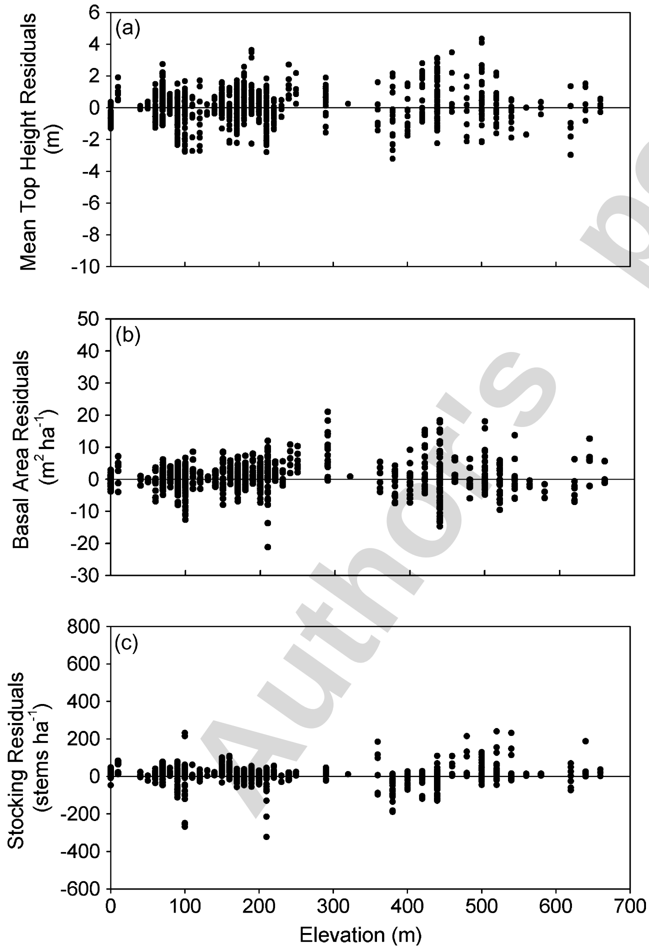

Fig. 4. Residual plots for CanSPBL(1.2) validation showing: (a) mean top height residuals vs. elevation, (b) basal area residuals vs. elevation, and (c) stocking residuals vs. elevation.

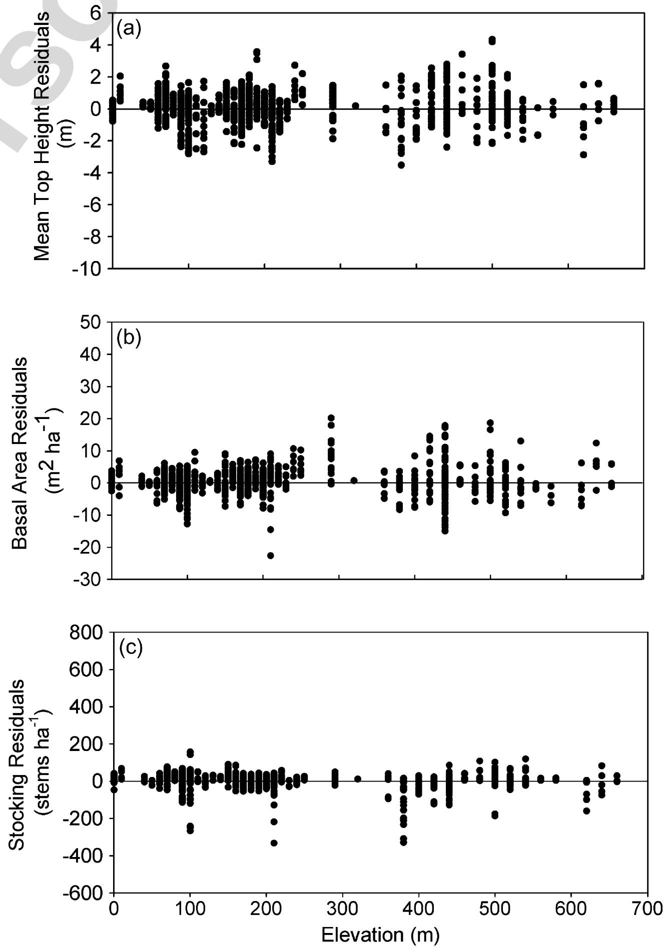

Fig. 5. Residual plots for CanSPBL(water) validation showing: (a) mean top height residuals vs. elevation, (b) basal area residuals vs. elevation, and (c) stocking residuals vs. elevation.

Table 4
Model residual distribution statistics for CanSPBL(1.2), CANTY, 3-PG, and CanSPBL(water) in terms of skewness, and kurtosis

Model	Residual distribution statistics	
	Skewness	Kurtosis
CanSPBL(1.2)		
Stocking	−1.01	11.01
Basal area	0.42	3.72
Mean top height	0.13	0.95
CANTY		
Stocking	−2.59	12.23
Basal area	−2.52	23.90
Mean top height	−0.29	0.95
3-PG		
Stocking	2.54	23.84
Basal area	1.61	5.07
CanSPBL(water)		
Stocking	−2.79	13.97
Basal area	0.22	4.04
Mean top height	−0.02	1.04

3.5. Comparison of model accuracy

Model validation statistics for each tested model component are listed in terms of mean square error, average model bias, and model efficiency in Table 3. Model residual distribution statistics for each tested model component in terms of skewness, and kurtosis.

Overall, the models CanSPBL(water), and CanSPBL(1.2) performed the best in terms of basal area and mean top height prediction. Both models CanSPBL(water), and CanSPBL(1.2) showed a slightly worse fit in predictions of stocking than did the model CANTY (Table 3). The hybrid model 3-PG showed a better fit for the prediction of basal area than the statistically based model CANTY, but showed a worse fit for the prediction of final stocking than all other models (Table 3).

In terms of distribution of residuals, CanSPBL(1.2) and CanSPBL(water) performed best in terms of skewness. On average kurtosis was lowest for CanSPBL(1.2), followed by CanSPBL(water) (Table 4). 3-PG performed the worst on average, in terms of the distribution of residuals. Correlation between model residuals and interval was also calculated to see if model error was related to re-measurement interval. The average correlation coefficient R for all models was −0.0027, which would indicate interval did not have a serious effect on model residuals.

4. Discussion

This paper quantitatively illustrates the utility of process-based modelling, and the accuracy of non-linear least-squares regression growth and yield models to predict forest growth. Interesting results of this study show that the process-based model 3-PG predicted basal area more precisely than a statistically based (state-space) model CANTY even though stem numbers predictions were less precise. This is an interesting result as it shows either CANTY's inability to deal with changing growing conditions at different elevations, or it shows possible changes in growing environment from the time of initial model fitting such as nutrient losses in the soil or perhaps changing climate. 3-PG also predicted basal areas that were comparable to those predicted by the models CanSPBL(-water), and CanSPBL(1.2). This result shows that process-based models can give comparable results for basal area prediction as statistically based models while maintaining the ability to make predictions under changing environmental conditions, such as varying climate and soil type. These results are also similar to those found by Tickle et al. (2001) when 3-PG proved more accurate than an older growth and yield model in Australian eucalypt plantations. The model 3-PG also has many physiological outputs such as estimated carbon pools, and leaf area that can be of use to forest managers interested in carbon accounting.

3-PG did not perform as well for the prediction of stocking compared to the other models tested. This effect may have been due to 3-PG's use solely of the −3/2 power law to predict mortality events. The −3/2 power law would only predict mortality events due to overcrowding and would be of little use in cases of mortality caused by wind, or insect attacks. Further, basal areas output from 3-PG are determined by inverting the allometric equation that describes the relationship between individual stem mass and diameter, calculating average basal area per tree and then multiplying by stem number, so if predicted stem numbers are inaccurate, then basal area will also be affected.

The statistical hybrid model CanSPBL(water) predicted basal areas and mean top height more accurately than all other models compared in the study but showed only modest gains in accuracy to the completely statistical model CanSPBL(1.2). These gains in accuracy did not happen solely on sites where water may have been limiting growth such as those located on elevations of 10–250 m, and may not warrant the extra data inputs needed to run the water balance version of the model.

The state-space model CANTY performed the worst in terms of basal area and mean top height prediction. This may have been a result of the critical dependence of state-space models on site index and that the height model is used instead of time when predicting basal area and stocking. If the height model is biased then the others are also likely to be biased. As the model CANTY was fitted to data of only short re-measurement intervals and lower elevation sites on the Canterbury plains, the model did not perform well predicting mean top height at elevations and at longer intervals. Site index is calculated as mean top height of the stand at age 20, so the problem with fitting mean top height can be compounded when trying to predict basal area with site indices that were originally biased. Residual errors in prediction of mean top height and basal area may also have been due to some fundamental problem in model construction as indicated by CANTY's inability to make unbiased predictions at even lower elevations (Fig. 2a and b). A better formulated empirical model may avoid these problems, though a thorough discussion of model formulation may be beyond the scope of this study.

The study shows that empirical growth models can often handle elevation gradients and the changing growing conditions that accompany them reasonably well within a region. The practice of incorporating the effects of elevation into empirical growth models in New Zealand has been used by Zhao (1999), and Pinjuv (2006). Using primary driving variables other than elevation, such as solar radiation, rainfall, and soil properties, may increase the range at which these models may be applied and allow model users to look at what if scenarios of changing climate and soil properties. The incorporation of these variables can also add to model complexity and the number parameter inputs (Table 1), and may make the models unwieldy for some practical applications.

5. Conclusions

Work presented in this paper highlights areas of strength and weakness for a range of modelling approaches in prediction of forest productivity. This paper demonstrates that a hybrid version of a statistical difference equation model (CanSPBL(water)) can provide the most accurate predictions of forest growth when compared to a range of model types. This model also maintains the ability to deal with changing growing and climatic conditions, though increases in accuracy are modest when compared to a statistical difference model (CanSPBL(1.2)). This paper also demonstrates that a simple process-based growth model (3-PG) can provide relevant forest productivity estimates of basal area more accurately than an older statistical state-space model (CANTY), which may indicate the expiration date of models of this type, in that it lacks the ability to make predictions under an environmental gradient and changing environmental conditions.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.foreco.2006.06.025.

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